Flexible Probabilistic Modeling of Astrophysical Dynamic Spectra

Thomas J. Loredo,[1] Carlo Graziani,[2] Jon Hakkila,[3] Robert Wolpert[4]

[1]Dept. of Astronomy, Cornell University; [2]Dept. of Astronomy, University of Chicago;

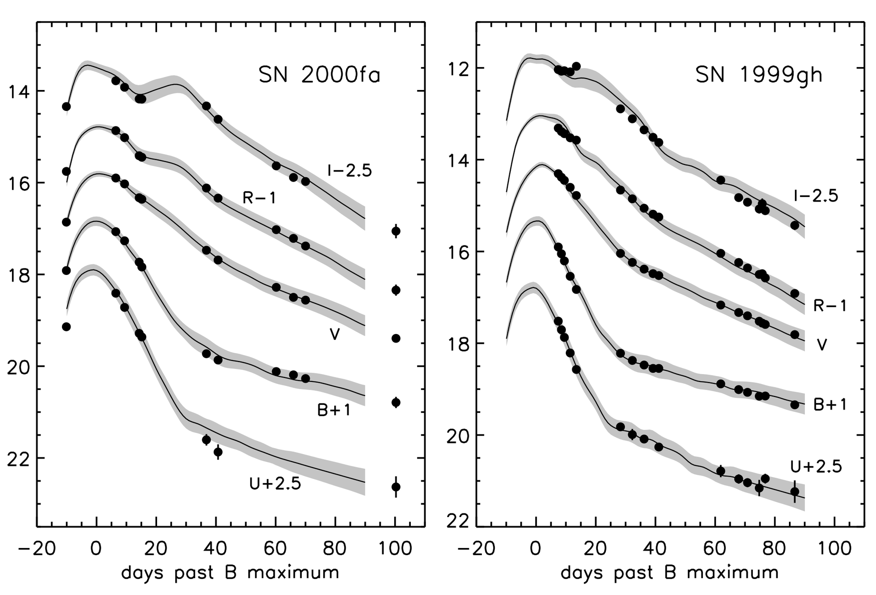
[3]Dept. of Physics &Astronomy, College of Charleston; [4]Dept. of Statistical Sciences, Duke University

Overview

The prompt emission from a gamma-ray burst (GRB) showers instruments with an extremely complex rain of hard X-ray and gamma-ray photons, varying wildly in rate and energy over time scales of milliseconds to hours. The months-long rise and decline of emission from a Type Ia supernova (SN Ia) presents telescopes with a complex, time-varying spectrum whose detailed behavior encodes information about its intrinsic luminosity, enabling use of SN Ia as cosmological distance indicators. What these two classes of sources have in common is that essential physics is hidden in a dynamical spectrum, an increasingly important astrophysical data type. We describe a project, newly-supported by the AISR program, developing new methods for analyzing complex dynamic spectra using semiparametric Bayesian models that describe the time-energy behavior as a two-dimensional stochastic process. We will analyze GRB emission using Lévy adaptive regression kernel models, which are well-suited to modeling signals resembling superpositions of many overlapping pulses with a spectrum of properties. For SN Ia, we will use multivariate Gaussian processes to analyze multicolor light curves; these are well-suited to modeling signals with smooth variation on a variety of scales. We describe properties of the models, and the impact successful modeling will provide on our understanding GRB physics and our ability to use SN Ia for precision cosmology.

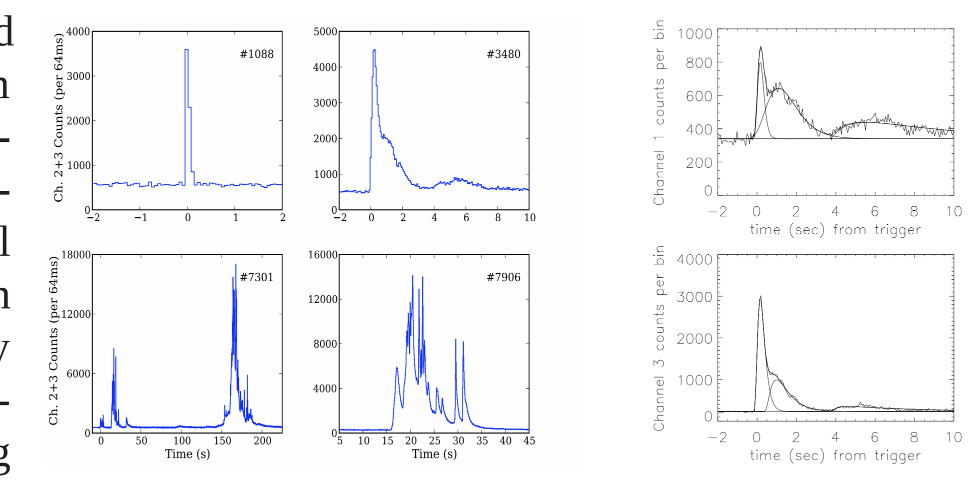
Two Motivating Problems

Type Ia Supernova Light Curves



The figure (from Jha, Riess & Kirshner 2007) shows simultaneous fits to SN Ia multicolor light curves (magnitude vs. time). *The amplitudes are not free parameters*; they are inferred from how the *shapes* (time scales) of the curves compare to a model trained on nearby SNe with known luminosities. This kind of model *calibrates SN Ia luminosities* enabling them to be used as distance indicators; this underpinned the discovery of *dark energy*. Systematic errors in current methods are much too large to enable accurate measurement of the dark energy equation of state with future surveys (e.g., using *JDEM*).

Gamma-Ray Burst Prompt Emission

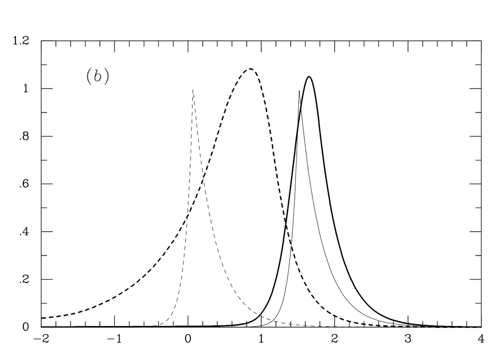


Prompt γ-ray and hard X-ray emission from GRBs provides a window into the still-puzzling GRB central engine. Also, it can be seen from very high redshift, making GRBs promising cosmological probes. The blue light curves (binned counts vs. time) display some of the diversity of GRB prompt emission; there are typically *many diverse pulses* (often overlapping), with a possible smooth component. The right curves show pulse fits to #3480 (Hakkila^{+} 2008); such fits quantify the diversity of energy-dependent pulse structures *within a single burst* (e.g., low energy lags high by times varying from ms to seconds). Current methods cannot fit crowded pulses and do not fully account for instrumental effects.

Semiparametric Dynamic Spectrum Modeling

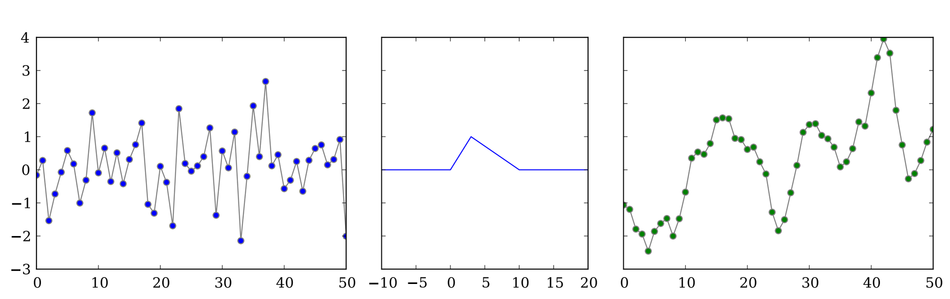
These problems share important features:

- A *dynamic spectrum* must be modeled that is too complex to admit a simple parametric description.
- *Instrumental and observational effects* (bandpasses; dust and extinction) must be carefully accounted for to enable accurate inference of the intrinsic emission.



We address these problems via *semiparametric Bayesian modeling*. We calculate a *posterior distribution* for the dynamic spectrum, specified directly and flexibly as flux or luminosity vs. energy and time (with additional finite parameters such as source redshift, luminosity, or extinction coefficients). It is proportional to a likelihood function (probability for the data given the model) that accounts for instrumental and observational effects via **forward modeling of the data**.

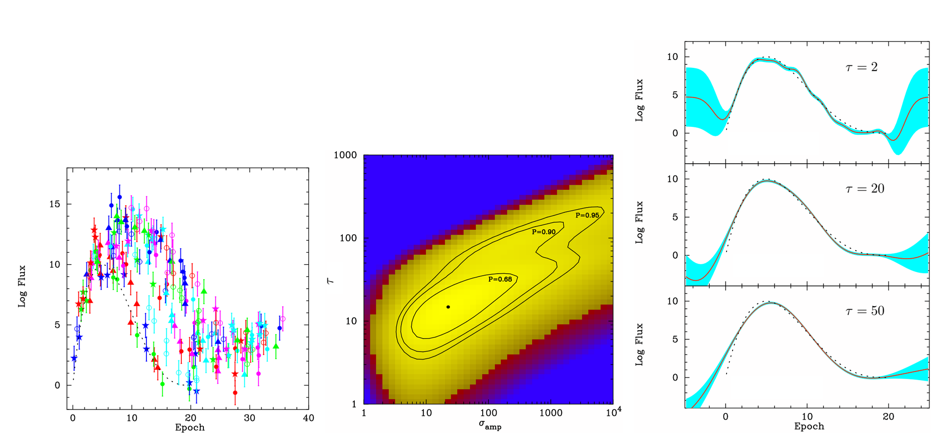
The figure above illustrates the importance of forward modeling for GRB pulse modeling. Thin curves show a simulated fast rise, exponential decay (FRED) pulse model with energy-dependent lag (dashed is at the nominal energy for BATSE ch. 3 (193 keV); solid for ch. 1 (48 keV)). Thick curves show the forward-folded signals, displaying distortions due to instrumental effects.



The posterior distribution is also proportional to a prior distribution that uses a **stochastic process** to control the flexibility of the model in a manner that adapts to the information in the data. The figure at right motivates our choices via a simple moving average process in discrete time. Left panel shows a latent time series of *independent, identically distributed* (IID) samples, here standard normal. Middle panel shows a *kernel function* defining a moving average. Right panel shows the visible time series from convolving the IID samples with the kernel. It is smoother than the latent series; samples follow a *correlated multivariate normal distribution* (MVN). Our SN Ia and GRB models generalize this idea to the continuum in two different directions.

Gaussian Processes for SN Ia

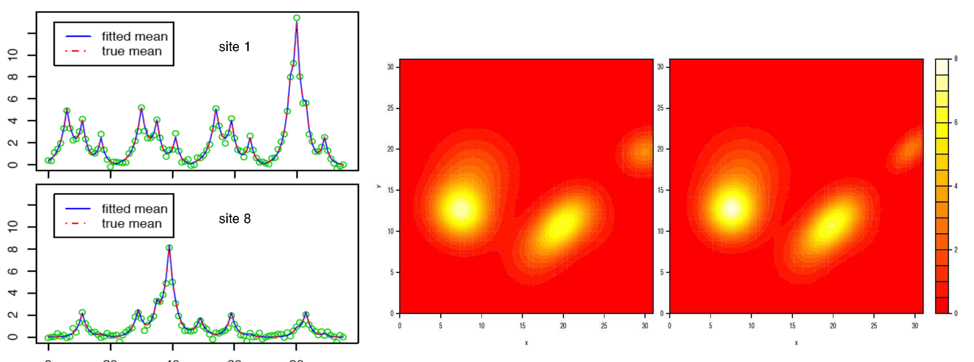
To model the smooth multicolor light curves of SN Ia, we will use **Gaussian process (GP) priors**. These extend the MVN idea from the moving average example to the continuum, replacing the discrete covariance matrix with a continuous *covariance function*, whose amplitude and scale control the variability and smoothness of the model in a way that adapts to the data.



The figure at right shows the approach at work in the simplified setting of single-color light curves with no extinction or cosmological effects. Left panel shows twenty simulated light curves with a SN Ia-like stretch-brightness relation built in; colors distinguish the sources. Middle panel shows contours of constant likelihood in the two GP “hyperparameters” controlling variability (σ) and temporal smoothness (τ). Dot shows best-fit values. Right panel shows the resulting fits for the predicted unstretched light curve that generated the data, vs. choice of τ, illustrating adaptivity. We are working to extend this to multicolor data, incorporating relevant observational effects.

Lévy Processes for GRBs

To model the multi-pulse dynamic spectra of GRBs, we will use **Lévy Adaptive Regression Kernel** (LARK) models. These extend the IID latent process and kernel ideas from the moving average example to the continuum. Generalizing IID, we require the latent process to have: (1) a joint distribution for total flux in disjoint intervals to be independent; (2) the distribution to be invariant to translation. This implies the latent process is a *Lévy process*. An analytical description is intricate, but there is a simple generative description involving strewing discrete point masses in the energy-time plane. LARK models add the kernel idea by including kernel parameters in the “strewing,” enabling kernel features to adjust to the data.



The figure, right, shows LARK modeling of simulated pollutant spatio-temporal monitoring data (Clyde^{+} 2008). Left shows measured (points) and true & fitted (curves) SO_2 levels vs. time (hours) at 2 of 33 randomly placed stations in a rectangular region; fits closely overlap truth for all stations. Right shows inferred (left panel) and true (right panel) spatial distributions at $t = 36$. We are adapting this approach to the GRB setting.